Navigation Physics: Journal of Physics Education
Vol. 7, No. 1, June 2025
E-ISSN: 2722-5593

Modelling the Tunneling in 1D Schrodinger Equation Using Graphical User Interface

Alpi Mahisha Nugraha[1] and Nurullaeli[1]
[1]Department of Informatics Engineering, Universitas Indraprasta PGRI, Jakarta, Indonesia

Abstract

Quantum tunneling is a phenomenon where particles have a probability of penetrating a potential barrier despite having total energy llower than the barrier height. This study analyzes the tunneling effect by solving the one-dimensional (1D) Schrodinger equation using the finite difference method to obtain the wave function evolution for various potential barrier configurations. The solution is implemented in a Graphical User Interface (GUI) MATLAB to facilitate analysis and visualization, allowing users to interactively adjust potential parameters, energy, and other conditions. Simulation results demonstratehow transmission probability depends on energy, height and width of potential barrier. This GUI provides an intiuitive tool for exploring quantun tunneling, making it valuable for both education and research in quantum physics.

Keywords: *Tunneling effect, Schrodinger Equations, GUI.*

Corresponding Author:
Alpi Mahisha Nugraha
Department of Informatics Engineering, Universitas Indraprasta PGRI, Indonesia
Jl. Raya Tengah No.80, RT.6/RW.1, Gedong, Kec. Ps. Rebo, Kota Jakarta Timur, Daerah Khusus Ibukota Jakarta 13760
Email: alpi.mahisha@gmail.com

1. INTRODUCTION

Despite its central role in quantum mechanics, the Schrödinger equation remains difficult for many students to grasp due to its abstract mathematical structure and the non-intuitive nature of the quantum phenomena it describes. This ongoing challenge underscores the importance of developing more accessible and engaging learning tools that can bridge conceptual understanding with visualization. To contextualize this need, it is helpful to reflect on the intellectual foundations of quantum theory. The 19th century often regarded as the golden age of physics was a period marked by profound theoretical advancements that reshaped our understanding of nature at atomic and subatomic scales. Among these developments was the emergence of wave particle duality, a revolutionary concept that challenged classical mechanics and paved the way for the formulation of the Schrödinger equation. Recognizing this historical trajectory emphasizes not only the significance of the equation itself but also the necessity of innovative educational approaches to make its implications more accessible to learners. In its one-dimensional form, depending only on position x, the Schrödinger equation is expressed as:

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x) + V(x)\psi(x) = E\psi(x) \tag{i}$$

with
$\hbar$ as the reduction of planck constant $\hbar = h/2\pi$,
m as mass of particle,
∇^2 as the Laplace operator representing the kinetic energy of a particle with mass m,
$\psi(x)$ as a wave function that contains probabilistic information about the system,
$V(x)$ as a potential energy that is a function of position,
and E is the eigenvalue that represents the energy level of the system.

The Schrödinger equation plays a central role in describing the behavior of particles. It encodes the probability distribution for locating a particle at a given position or time, allows for the analysis of quantum superposition where a particle can exist in a combination of states simultaneously and captures other fundamental quantum effects. Typically, this equation is introduced in greater detail at the university level, particularly in physics or applied physics programs. Despite its wide-ranging applications, especially in advancing modern technology, many students still find it one of the more difficult and daunting topics in their studies.

The modeling of the Schrödinger equation is typically introduced in quantum physics courses, where students are often challenged by its abstract formalism and the non-intuitive behavior it describes. To address this, various instructional modules have been developed to enhance engagement and conceptual clarity. Tools such as the 3D PageFlip electronic module (Yuli Yanti et al., 2017), supersymmetry-based energy spectrum visualizations (Saregar, 2015), virtual laboratories (Manurung et al., 2018), and interactive simulations using Macromedia Flash MX (Yuliani, 2017) have demonstrated potential in making quantum concepts more approachable. These tools are particularly valuable in visualizing phenomena that lack classical analogs, such as quantum tunneling, by providing students with graphical representations that bridge the gap between mathematical formalism and physical intuition.

However, many of these approaches offer limited interactivity or rely on predefined content, restricting learners' ability to explore parameter-dependent behavior dynamically. In contrast, GUI-based tools such as those built using MATLAB's Graphical User Interface offer a more flexible and responsive environment where users can manipulate potential barriers, energy levels, and other variables in real time. This interactivity is especially advantageous for helping students conceptualize quantum tunneling, where the probabilistic penetration of particles through potential barriers defies classical expectations. By visualizing the wavefunction's behavior under different scenarios, students are more likely to develop an intuitive grasp of the underlying physics. Accordingly, this study proposes the development of a GUI-based numerical solver for the Schrödinger equation as a pedagogical tool designed to enhance both engagement and conceptual understanding in quantum physics instruction.

Solving the Schrödinger equation is by no means a new endeavor. Some approaches rely on analytical methods to determine the corresponding wave functions (Dinnullah, 2015), and in certain cases, the equation has been applied to calculate the classical mass of stable nonlinear waves, known as solitons, that propagate without changing shape due to a balance between dispersion and nonlinearity in the medium (Prayitno, 2011). However, analytical solutions are often mathematically complex, making it difficult for students to grasp both the solutions and the physical meaning underlying the Schrödinger equation.

2. METHODS

The Schrödinger equation can be solved analytically in one dimension (1D), two dimensions (2D) (Supriadi et al., 2017), or in both space and time domains. However, analytical solutions become increasingly difficult to obtain as the modeled system grows more complex. In this study, the Schrödinger equation is solved using a numerical approach rather than an analytical one The finite difference method is used to discretize the one-dimensional Schrödinger equation as presented in Equation (i). The second derivative, in this context, can be approximated numerically as follows:

$$\frac{d^2\psi}{dx^2} \approx \frac{\psi_{i+1} - 2\psi_i + \psi_{i-1}}{\Delta x^2} \qquad \text{(ii)}$$

here $\Delta x = x_{i+1} - x_i$ and i denotes the index corresponding to discrete spatial positions. Based on this discretization, the time-independent Schrödinger equation can be expressed numerically as follows:

$$-\frac{\hbar^2}{2m\Delta x^2}\psi_{i-1} + \left(\frac{\hbar^2}{2m\Delta x^2} + V_i\right)\psi_i - \frac{\hbar^2}{2m\Delta x^2}\psi_{i+1} = E\psi_i \qquad \text{(iii)}$$

or it can be formulated in matrix form as follows:

$$\begin{pmatrix} a_1 & b & 0 & \dots & 0 \\ b & a_2 & b & \dots & 0 \\ 0 & b & a_3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & b & a_N \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_N \end{pmatrix} = E \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \vdots \\ \psi_N \end{pmatrix}$$

with: $b = \frac{\hbar^2}{2m\Delta x^2}$ and $a_i = \frac{\hbar^2}{2m\Delta x^2} + V_i(x)$. N represents the number of iterations performed in solving the matrix form of the equation. The solution must satisfy the normalization condition, meaning that the total probability of the wavefunction must equal one, expressed as $\int |\psi_i(x)|^2\, dx = 1$.

The numerical solution is presented through a MATLAB based Graphical User Interface (GUI), designed to assist students in understanding the Schrödinger equation particularly in illustrating quantum tunneling phenomena. In the designed GUI, users are required to input several parameters: the value of X, representing the length of the dimension; the energy and potential values in electron volts (eV); the width of the potential barrier; and the number of iterations used to sample the X values. The GUI then calculates and displays the transmission coefficient, which indicates the likelihood of the wave passing through the potential barrier. Additionally, the GUI generates a graph of the wave function $\psi(x)$ versus position x, along with its behavior relative to the energy and potential profiles, based on the input potential barrier width.

The workflow of the designed GUI is illustrated in Fig. 1. This study uses MATLAB as the programming language because it is both practical and powerful for solving equations using numerical methods. Moreover, the GUI is designed to remain user-friendly, ensuring that both students and learners can easily operate the 1D Schrödinger equation GUI developed in this work.

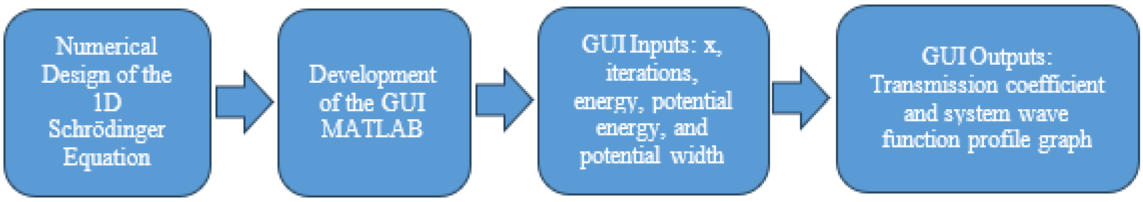

Figure 1. Research Workflow

3. RESULT AND DISCUSSION

The Schrödinger equation offers a fascinating framework for physical modeling, particularly because it enables the explanation of phenomena that are difficult to describe macroscopically but can be understood at the microscopic level. This equation can be solved analytically, numerically, or through a combination of both methods, each with its own advantages and limitations. Numerical approaches have been employed to make the physical phenomena described by the Schrödinger equation more accessible and easier to visualize. Examples include numerical solutions involving non-central Coulombic Rosen-Morse potentials (Yanuarief & Al-Faruq, 2019), the calculation of electron energy eigenvalues in finite potential wells (Luba et al., 2021), and the Dirac particle in a dynamic potential well (Tiandho, 2016). However, these solutions are often limited to numerical simulations, making them less accessible to students and learners who may not have the tools or background to engage with such material directly.

One accessible medium for students is the use of a Graphical User Interface (GUI) as an alternative learning tool. In addition to its visually appealing layout, the interface is user-friendly, making it easy to compute solutions to the Schrödinger equation. In this study, the GUI is designed with an initial interface as shown in Figure 2. Users, whether students or university learners, are only required to input values based on the conditions they wish to explore, particularly to observe the tunneling phenomenon. The required inputs include the value of X, representing the position range over which the wave function is analyzed; the number of iterations, which determines the resolution of the wave function the more iterations, the smoother the result; the energy value, or eigenvalue of the wave function, in electron volts (eV); the potential barrier, also in eV, which represents the obstacle the wave must overcome; and the width of the potential barrier, which influences how much of the wave is able to tunnel through.

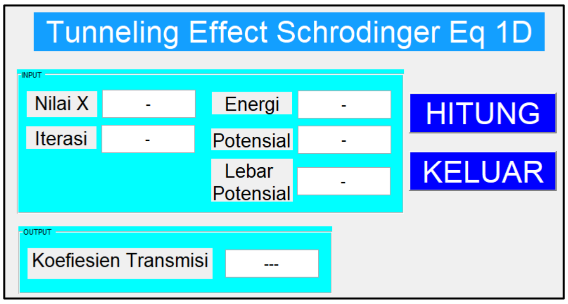

Figure 2. Initial GUI MATLAB

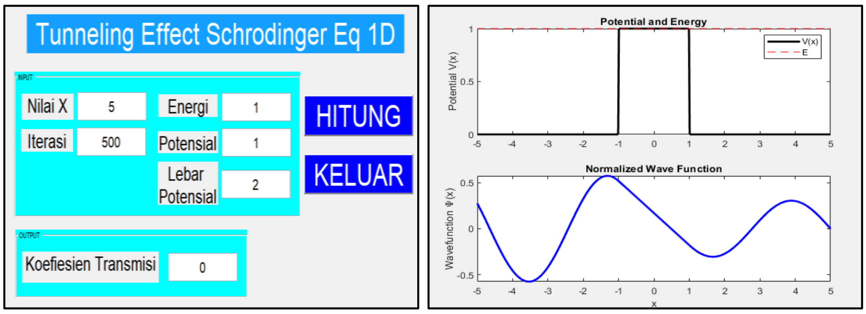

Figure 3. Input (Left) and Output (Right) in Energy = Potential energy

The output of the designed GUI is shown in Figures 3, 4, and 5. In addition to displaying the transmission coefficient which indicates the likelihood of a wave undergoing tunneling the GUI also presents the wave function profile as a function of position x. In Figure 3 (left), with input parameters x = 5, 500 iterations, energy of 1 eV, potential of 1 eV, and a potential barrier width of 2 spatial units, the transmission coefficient is found to be 0. This result indicates that when the wave's energy is equal to the height of the potential barrier, no transmission occurs, and tunneling is effectively prohibited. Nevertheless, as shown in Figure 3 (right), the wave function profile is still visibly disturbed in the region between -2 and 2, which corresponds to the location of the potential barrier.

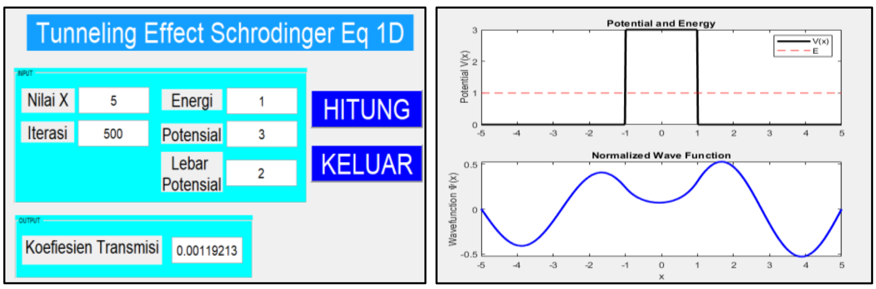

Figure 4. Input (Left) and Output (Right) in Energy < Potential energy

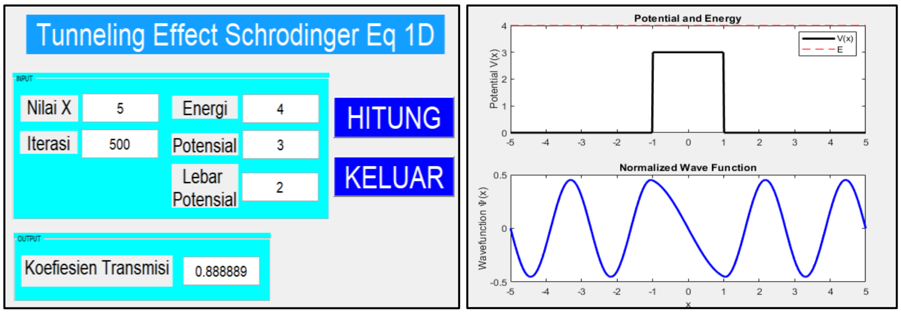

Figure 5. Input (Left) and Output (Right) in Energy > Potential energy

The tunneling effect is demonstrated in Figure 4, where the wave energy is set to 1 eV significantly lower than the potential barrier height of 3 eV. Despite this energy disparity, the transmission coefficient is approximately 0.0012, indicating a small but nonzero probability of the wavefunction penetrating the barrier. This exemplifies the quintessential quantum behavior that defies classical intuition, in which a particle lacking sufficient energy to surmount a barrier can still probabilistically "tunnel" through it. Such behavior is not merely theoretical; it underpins real-world phenomena such as alpha particle decay in radioactive nuclei, where particles escape the nuclear potential well despite having insufficient classical energy. The same principle governs the operation of tunnel diodes, where electrons quantum mechanically tunnel through a potential junction, allowing for high-speed switching in modern electronic circuits.

In contrast, Figure 5 presents a scenario where the wave energy (4 eV) exceeds the barrier height (3 eV). Here, the transmission coefficient rises sharply to approximately 0.89, reflecting a dramatic increase in tunneling probability. However, full transmission is not achieved, demonstrating that even when the energy surpasses the barrier, reflection still occurs due to quantum interference effects at the potential boundaries. This partial transmission is characteristic of quantum wave behavior and highlights the probabilistic nature of quantum transport. The comparison between these two cases illustrates how tunneling probability is sensitive not only to the relative energies but also to the structure and width of the potential barrier. Such insights, supported visually through the GUI-based simulation, provide learners with an intuitive yet rigorous understanding of quantum dynamics bridging abstract theory with observable physical behavior.

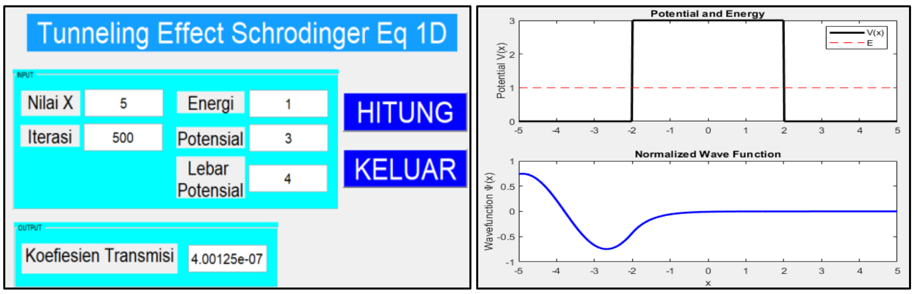

Figure 6. Input (Left) and Output (Right) or Varying Potential Barrier Widths

In Figure 6, using the same input parameters as in Figure 4 where the wave energy (1 eV) is lower than the potential barrier height (3 eV) the width of the barrier is doubled compared to the configuration in Figure 3. This adjustment leads to a significantly lower transmission coefficient of approximately 4 $\times 10^{-7}$, a sharp decrease from the previous value of 0.0012. This notable reduction illustrates how sensitive the tunneling probability is to changes in barrier width. A basic correlation analysis indicates that doubling the width results in an exponential decline in the tunneling probability, roughly by a factor of 10^3.

This trend aligns well with the prediction of the WKB (Wentzel–Kramers–Brillouin) approximation (Takada & Nakamura, 1994), which models the transmission probability T for a rectangular barrier as $T \approx e^{-2ka}$ where $k = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$ here, a represents the width of the potential barrier, V_0 is the barrier height, and E is the energy of the particle. Since T depends exponentially on a, even modest increases in width can lead to dramatic reductions in tunneling probability. The simulation results in Figure 6 thus not only provide a visual confirmation of this quantum behavior but also offer an intuitive way for learners to connect theoretical predictions with observable outcomes.

Based on the simulations conducted in this study, the transmission coefficient of a wave encountering a potential barrier is influenced by the wave's total energy, as well as the height and width of the potential barrier. The use of MATLAB's GUI provides users with a more accessible and intuitive way to analyze the tunneling effect a quantum phenomenon in which particles penetrate potential barriers. The designed GUI is expected to serve as an engaging learning alternative for students and university learners in studying Quantum Physics, particularly in solving the Schrödinger equation and visualizing the tunneling phenomenon more effectively.

4. CONCLUSION

This study successfully analyzed the quantum tunneling effect by solving the one-dimensional Schrödinger equation using the finite difference numerical method. The implementation through a MATLAB-based Graphical User Interface (GUI) enables more intuitive visualization and allows users to interactively explore system parameters. Simulation results show that the transmission coefficient, or transmission probability, depends on the particle's energy as well as the height and width of the potential barrier, which aligns with quantum mechanical theory. Thus, solving the Schrödinger equation via the GUI offers an effective alternative tool for understanding tunneling phenomena.

Future work could focus on improving numerical accuracy by employing more precise methods than finite difference, extending the model to higher dimensions beyond just the position x, and incorporating more complex spatial modeling. Additionally, integrating machine learning techniques could provide further insight into tunneling and other quantum physical phenomena.

REFERENCES

Dinnullah, R. N. I. (2015). Solusi Eksak Gelombang Soliton: Persamaan Schrodinger Nonlinear Nonlokal (NNLS). *Cauchy*, *3*(4), 39–44.

Luba, C. S. U., Warsito, A., & Pingak, R. K. (2021). *Kajian Komputasi Numerik Energi Eigen Elektron dalam Sumur Potensial Berhingga* (Vol. 6, Issue 1).

Manurung, S. R., Harahap, M. B., Rustaman, N. Y., & Brotosiswoyo, B. S. (2018). Implementasi Laboratorium Virtual dalam Pembelajaran Pendahuluan Fisika Kuantum untuk Meningkatkan Kemahiran Generik. *Jurnal Kependidikan*, *2*(2), 382–394.

Prayitno, T. B. (2011). Massa Klasik Soliton Persamaan Schrodinger Nonlinear. *Berkala Fisika*, *14*(3), 75–80.

Saregar, A. (2015). Analisis Spektrum Energi dan Fungsi Gelombang Potensial Non-Central Menggunakan Supersimetri Mekanika Kuantum. *Jurnal Ilmiah Pendidikan Fisika Al-Biruni*, *4*(2), 193–203. https://doi.org/10.24042/jpifalbiruni.v4i2.92

Supriadi, B., Prastowo, H. B., Amrullah, A. F., & Ridlo, Z. R. (2017). *Solusi efek terobosan penghalang ganda dengan persamaan Schrödinger dua dimensi*.

Takada, S., & Nakamura, H. (1994). Wentzel-Kramers-Brillouin theory of multidimensional tunneling: General theory for energy splitting. *The Journal of Chemical Physics*, *100*(1), 98–113. https://doi.org/10.1063/1.466899

Tiandho, Y. (2016). Partikel Dirac dalam Sumur Potensial Dinamis. In *Jurnal EduMatSains* (Vol. 1, Issue 1).

Yanuarief, C., & Al-Faruq, A. (2019). Solusi Metode Numerik Beda Hingga pada Visualisasi Fungsi Gelombang Persamaan Schrodinger Potensial Non Sentral Coulombic Rosen Morse. *Sunan Kalijaga Journal*, *1*, 28–36.

Yuli Yanti, R., Aminoto, T., & Berthalita Pujaningsih, F. (2017). Pengembangan Modul Elektronik Menggunakan 3D PageFlip Professional Materi Atom Hidrogen Pada Mata Kuliah Fisika Kuantum. *JurnalEduFisika*, *02*(01), 13–24.

Yuliani, H. (2017). Pembelajaran Fisika menggunakan Media Animasi Macromedia Flash-MX dan Gambar untuk Meningkatkan Pemahaman Konsep Mahasiswa. *Jurnal Ilmiah Pendidikan Fisika Al-Biruni*, *6*(1), 13–21. https://doi.org/10.24042/jpifalbiruni.v6i1.596

BIOGRAPHIES OF AUTHORS

	**Alpi Mahisha Nugraha** is a Doctoral Candidate at the Department of Physics, Postgraduate School, Universitas Indonesia, and a Lecturer in the Department of Informatics Engineering at Universitas Indraprasta PGRI, Jakarta, Indonesia. His research interests include Nuclear Physics, Computational Physics, and physics education media. He can be reached via email at alpi.mahisha@gmail.com
	**Nurullaeli** is a lecturer in department of informatics engineering, Universitas Indraprasta PGRI, Jakarta, Indonesia. Her research focuses on biophysics, computational physics, and physics education media. She can be contacted at email: leli.biofisika@gmail.com